The University of Georgia

Mathematics Education
EMAT 4680/6680 Mathematics with Technology
Jim Wilson, Instructor

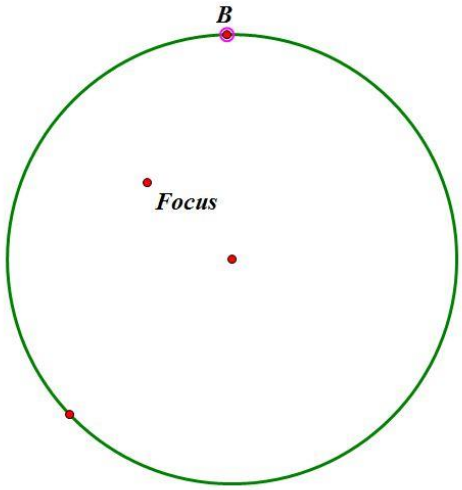
Constructions of the Ellipse and Hyperbola

By: Lindsey Harrison

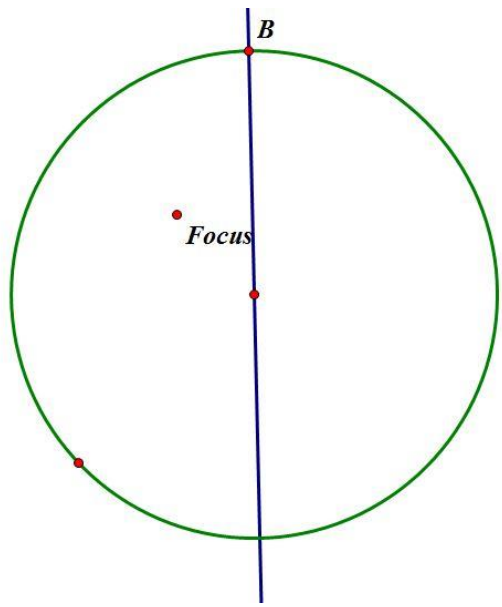
Objective: The goal of this exploration is to further explore variations of the focus-directrix construction in order to produce an ellipse and a hyperbola. For each, we will accomplish this by tracing a point of tangency, tracing the tangent line, and using the locus command on Geometer's Sketchpad.

Prompt: Construct the locus of points equidistant from a fixed point F and a circle. In other words, repeat the parabola construction, but use a circle as the "directrix". Let F be any point in the plane other than the center of the circle. Assume F is not on the circle; it can be either inside or outside.

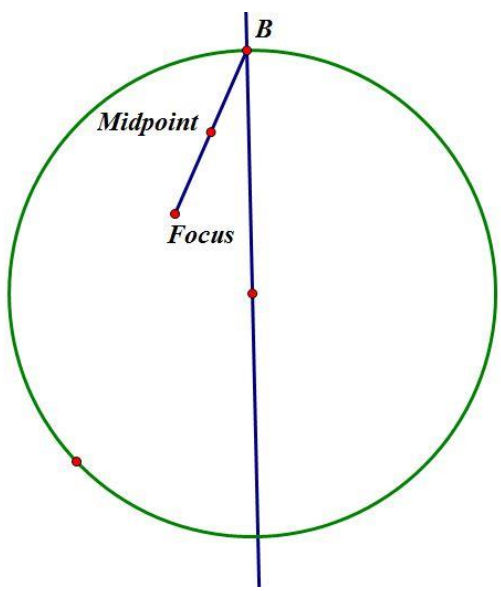
Ellipse: An ellipse is a set of points whose sum of their distances from two given points, called the foci, is constant. For the following construction we will start with a circle (our directrix), an arbitrary point B on the circle, and a focus point inside the circle. We will use the center of the circle as our other focus point.



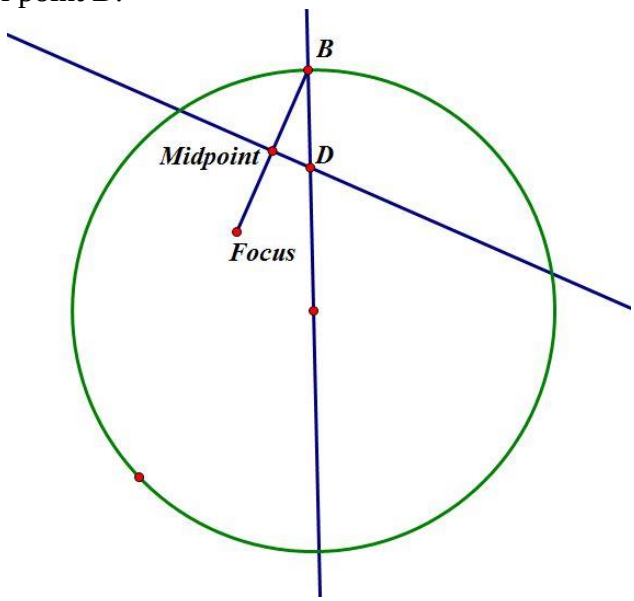
1. Construct a line through the center of the circle and point B on the circle.



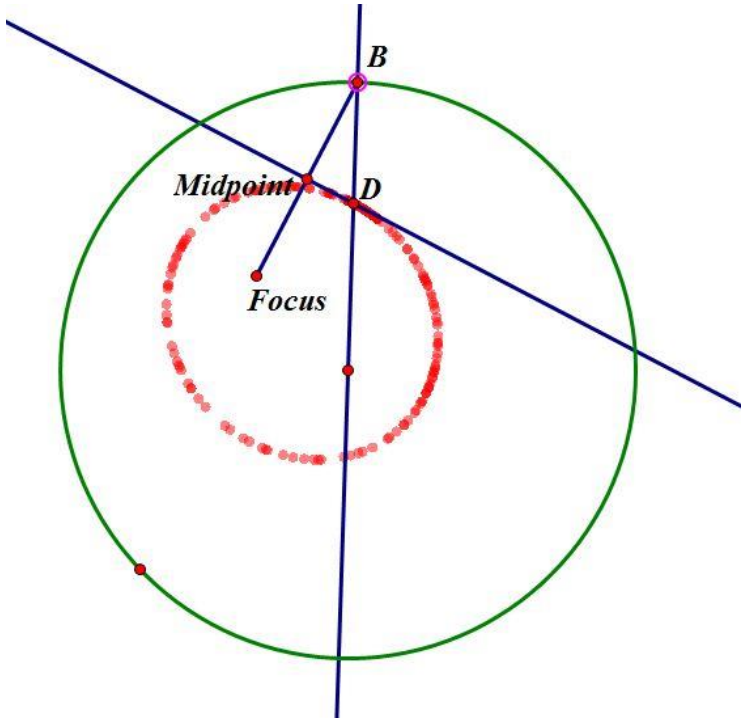
2. Next construct a segment from point B to the point labeled Focus. Mark the midpoint of this line segment, as it is equidistant from B and the focus labeled Focus.



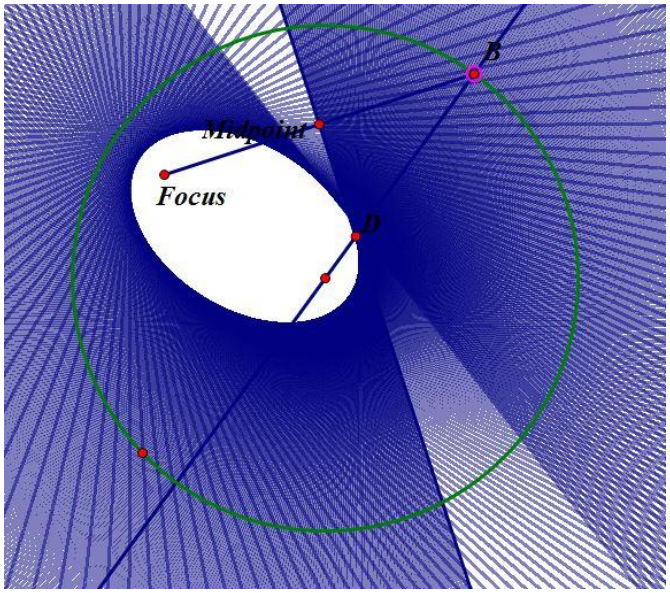
3. We then construct a perpendicular line to the segment connecting point B to Focus through the midpoint. Where this perpendicular line interests our original line, we label point D.



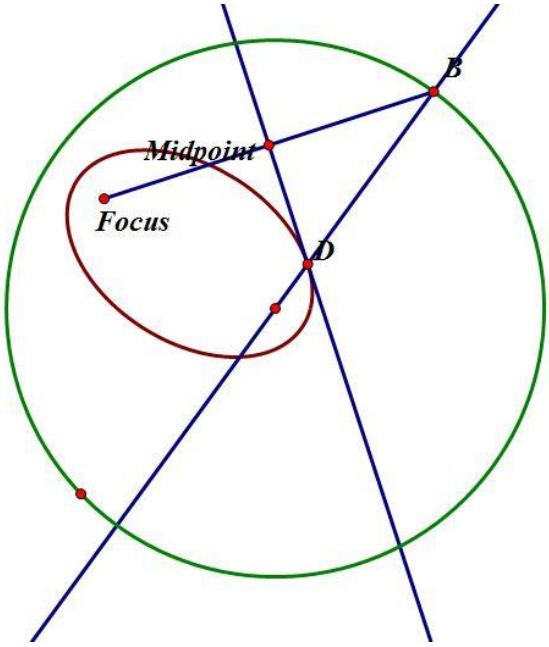
4. To outline our ellipse, trace point D and animate point B about the directrix (our circle).
   GSP FILE



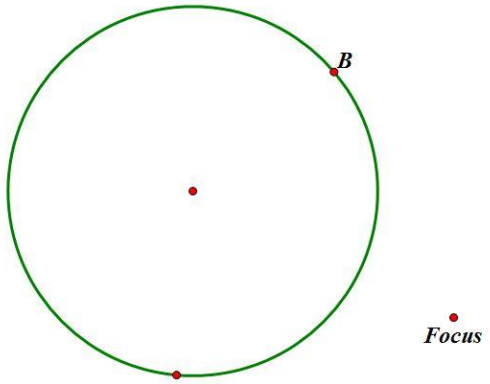
5. We can now repeat this construction through step 3 and trace the perpendicular line while we again animate point B.
   GSP FILE



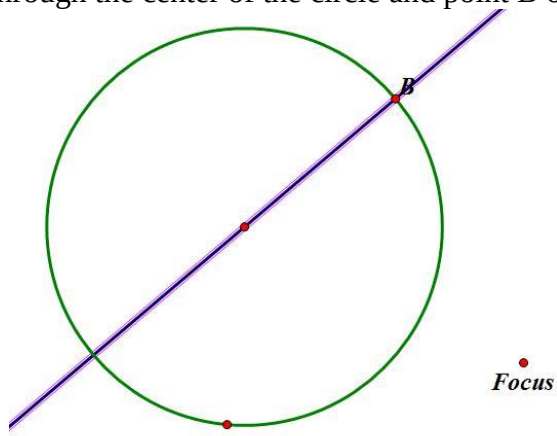
6. For the last ellipse construction we will use the locus command. The benefit to this method is that we then have an object which we can select and manipulate instead of just trace. Repeat the construction again through step 3. Select points D and B and then select the locus command.
   GSP FILE



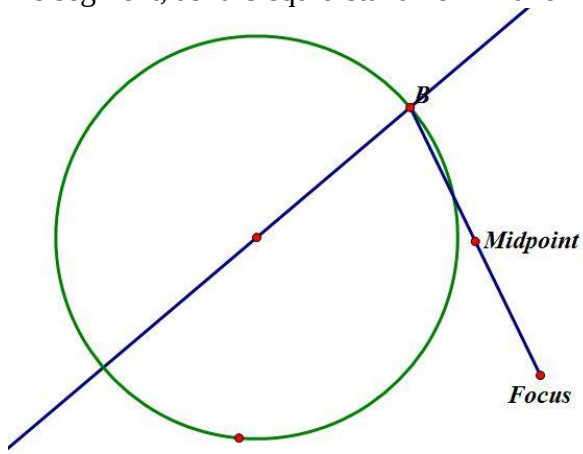
Hyperbola: A hyperbola is a set of points whose difference of their distances from two given points, called the foci, is constant. For the following construction we will start with a circle (our directrix), an arbitrary point B on the circle, and a focus point outside the circle. We will use the center of the circle as our other focus point.



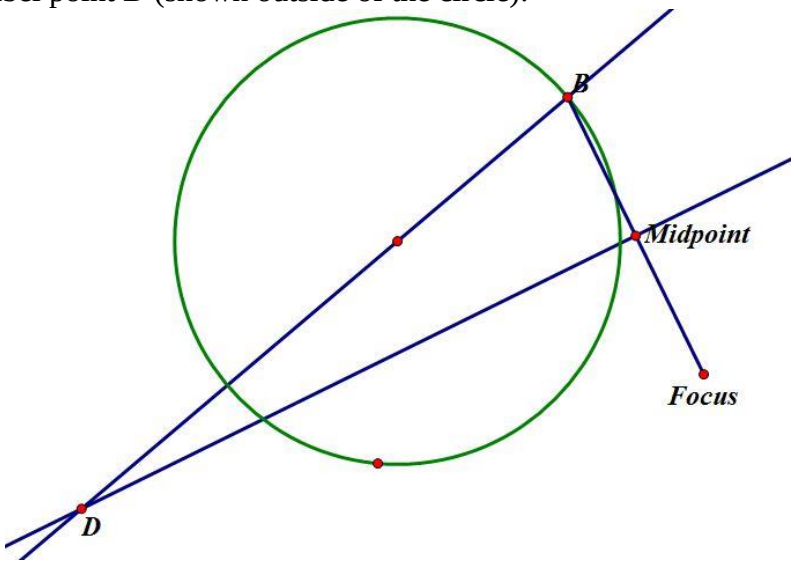
1. Construct a line through the center of the circle and point B on the circle.



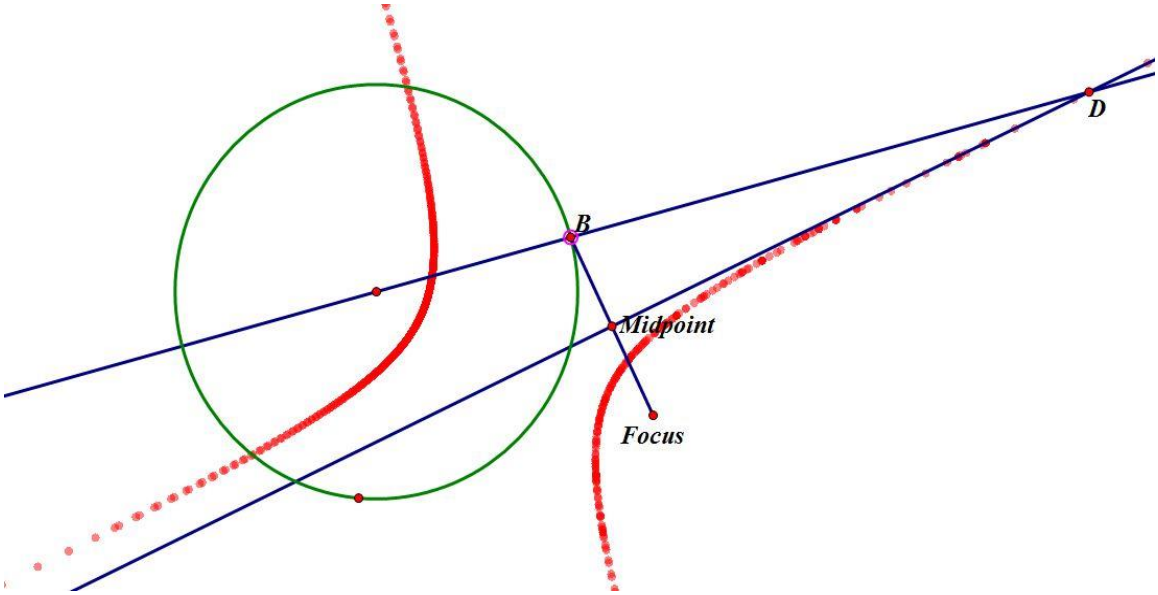
2. Next construct a segment from point B to the point labeled Focus. Mark the midpoint of this line segment, as it is equidistant from B and Focus.



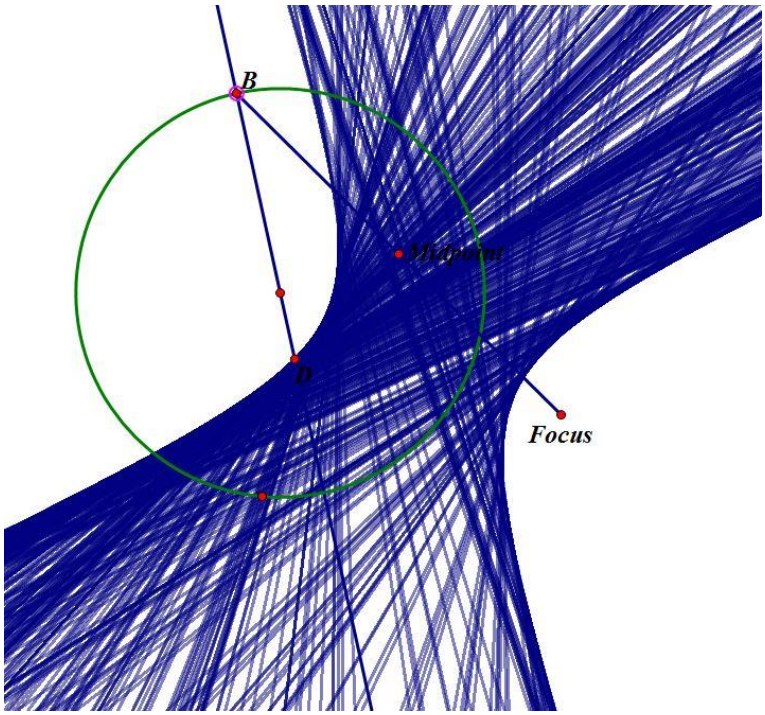
3. We then construct a perpendicular line to the segment connecting point B to Focus through the midpoint. Where this perpendicular line interests our original line, we label point D (shown outside of the circle).



4. To outline our hyperbola, trace point D and animate point B about the directrix (our original circle in green).
   GSP FILE



5. We can now repeat this construction through step 3 and trace the perpendicular line while we again animate point B.
   GSP FILE



6. For the last hyperbola construction we will use the locus command. Repeat the construction again through step 3. Select points D and B and then select the locus command.
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